Guidance paper - The use of calculators in the teaching and learning of mathematics

Background and context

In mathematics, the calculator can be an effective teaching and learning resource in the primary classroom. Calculators can be used with children in all age groups across the Foundation Stage, Key Stage 1 and Key Stage 2.

The revised Framework for mathematics includes objectives that promote the use of calculators as a tool for calculations. These objectives span Years 4, 5 and 6. However, in the context of exploring numbers and the number system, the calculator can be used with other age groups as a teaching and learning tool. This paper sets out to offer guidance on the use of calculators in the teaching and learning of mathematics.

In the primary mathematics classroom calculators can be used for different purposes by teachers and children. These include:

- teaching children how to use a calculator effectively to calculate and to recognise how and when it is appropriate to do so, by first deciding if mental and pencil-and-paper methods are quicker or more reliable
- supporting the teaching of mathematics where the aim is to focus on solving a problem rather than on the process of calculation
- providing a tool with which children can explore patterns in numbers and identify properties and relationships
- consolidating children's learning of number facts and calculation strategies.

Using calculators to support teaching and learning

Children are quite likely to see and use calculators or keypads of some sort in their home environment. In the Foundation Stage children become familiar with calculators, and how they can be used, through play. They use calculators in role-play such as the garden centre, the shop or the post office, and play with them to explore what the keys do. As children find out how to display familiar numbers, they also, through conversation with adults, learn to key in larger numbers. For example, a child posing the question: 'Where is the ten key?' can stimulate discussion on how the number 10 is formed.

By the end of Year 1 children are expected to be able to read and write numbers to 20 and beyond. Keying in two-digit numbers and reading them accurately is an activity that supports these skills. The entry of single digits to form two-digit numbers can strengthen children's knowledge of place value. Exploring numbers that are ten more or less than a given number or understanding doubles can be introduced and supported with a calculator. The calculator is being used to support children's thinking and recognition of the effects when numbers are increased by ten or doubled.

In Year 2 children are expected to describe patterns in numbers and derive and recall the 2, 5 and 10 times-tables. Using the constant facility on a calculator to add 2s, 5s and 10s repeatedly to 0 is a way to derive the multiples and can help children identify the pattern of the numbers in the sequence. The calculator can also be used to sharpen and secure mental calculation. In the table below, the numbers are generated by adding or subtracting a single-digit number.

3	7	2	8	10	9	5	3

The aim is to move along the track from one cell to another, using a calculator. Children first key in 3, then key in the correct operation to show the next number on the display, in this case using the operations [+] 4, [–] 5, [+] 6, etc. The calculation has to be carried out mentally before the calculator is used. The calculator provides a tool to confirm the calculations and the children also learn how to use the addition and subtraction keys to enter number sentences.

A key objective in Year 3 involves place value and the partitioning of three-digit numbers. Using the calculator can give children the opportunity to practise and apply their knowledge, skills and understanding. For example, the children key in a three-digit number, say 346. They have to eliminate the digits one by one and can only subtract multiples of 100, 10 and 1 to do so, with the largest multiple first. Subtracting 300, 40 and then 6 works, but what if the keys with the digits 3, 4 and 6 are barred? What is the quickest way to reduce 346 to 0 using multiples of 100, 10 and 1? What if the number 4643 is to be reduced to 0 and both 4s must be eliminated first in one step?

These are some of the calculator activities that can support mathematics learning to Year 3. With this kind of experience children should enter Year 4 confident at keying in numbers and number sentences that involve any of the four operations. For further ideas on how a calculator might be used across the years, see part 5 of *Teaching mental calculation strategies,* a joint NNS and QCA publication that is available from QCA

Publications, telephone 08700 606015.

The Statutory Curriculum

The use of a calculator as a tool to carry out calculations is embedded within the National Curriculum Key Stage 2 Programme of Study for Mathematics. The use of calculators is assessed in the statutory tests at the end of Key Stage 2. Evidence from the analysis of test scripts shows that children are unsure about when and how they might use the calculator. They are not clear about using calculator methods or recording their method.

Within the Key Stage 2 Programme of Study, Ma2 Number (see *Calculator methods*) states that children should be taught to:

use a calculator for calculations that involve several digits, including decimals

use a calculator to solve number problems

know how to enter and interpret money calculations and fractions

know how to select the correct key sequence for calculations with more than one operation.

It is important that children become confident users of calculators. They need to recognise that the calculator is a tool they are in control of and to understand how it can help them to develop their mathematics. The key messages are that:

- the teacher controls the children's access to and use of the calculator
- the calculator is not a prop for calculations that can and should be carried out mentally or with pencil and paper
- calculators should not be used as a tool to carry out calculations until Year 4, when children are expected to be able to add and subtract two-digit whole numbers mentally and to derive and recall multiplication facts and the corresponding division facts to 10 × 10.

The calculator as a tool to carry out calculations

If children are to be confident users of calculators, they need to be taught the basic skills of entering numbers and operations. They need to be able to interpret the values displayed both during this process of entering numbers and when they review their answers. They need to decide if the answer displayed is sensible and if it needs any adjustment to take account of rounding errors and to incorporate suppressed zeros. To use a calculator effectively requires a secure knowledge of number, which has to be the prime aim.

The revised Framework places an emphasis in Key Stage 1 and the first two years of Key Stage 2 on securing children's knowledge of number facts and mental calculation strategies. They also begin to develop written methods that they can apply more generally. In Year 4, children are expected to solve problems using calculator methods where appropriate. As children learn how to enter simple one-step calculations that involve whole numbers, they can explore the behaviour of the four operations and the properties of these numbers. They begin to recognise how their number knowledge can be applied to calculations that involve more complex operations with larger whole numbers.

Over the course of Years 5 and 6, children learn how to use other functions on the calculator and apply their skills and knowledge to decimal numbers, fractions and negative numbers. They solve multi-step problems and use the calculator to generate sequences of numbers and families of calculations. Children recognise underlying properties and principles that they can then apply when calculating mentally or on paper.

The general aim is that, by the end of Key Stage 2, children know how to transfer their knowledge and understanding of numbers and the four operations to mental, written and calculator methods of calculation. They can explain and record their methods in succinct and manageable ways.

More specifically, the aim is that when children leave primary schools they:

- have a secure knowledge of number facts and a good understanding of the four operations
- are able to use this knowledge and understanding to carry out calculations mentally and to apply both general strategies when using single-digit and two-digit numbers and particular strategies to special cases that involve larger numbers
- make use of jottings to help record steps and part answers when using mental methods that generate more information than they can keep in their heads
- use efficient written methods of calculation, which they can apply with confidence when undertaking calculations that they cannot carry out mentally
- use a calculator effectively, using their mental skills to monitor the process, to check the steps involved and to decide if the numbers displayed make sense.

Learning objectives

The objectives in the revised Framework make specific reference to calculators in the 'Using and applying mathematics' and 'Calculating' strands for Years 4, 5 and 6, and Year 6 progression to Year 7. These objectives are appended at the end of this paper for reference.

Below is an expanded set of learning objectives that cover Year 4 and upwards, which your school might use to review its policy and planned provision in the use of calculators in the teaching and learning of mathematics. These are divided into the technical knowledge and skills children need to use a calculator effectively, and the interpretive skills and understanding they need to apply these skills to support their learning. There is accompanying commentary on aspects of teaching and learning.

A. The technical knowledge and skills needed to use a calculator effectively

Recognise the operations that the keys on the calculator represent

Calculators can operate in different ways and the detail in the displays can vary. For example, some calculators display the full number sentence during a calculation, while others only display the number entered or the answer. Spend time becoming familiar with the calculator the children are to use. Be ready to show children how the different keys work (e.g. how the decimal point key is used). The decimal point may already appear on the display at the end of the number. After the key is pressed it appears to move with the number, which can confuse children. The [+/–] key changes the sign of the number displayed. It toggles between + and –. This can be used to highlight the difference between using + and – to represent the operations addition and subtraction, and to indicate whether the number is positive or negative. Children may miss the sign that indicates a number is negative when this appears on the extreme left of the display or after the number. Many calculators have a percentage key with the symbol %. Its use is likely to cause confusion and it is better not to use it with most primary children. Children need to be taught the essential features of the calculators available to them in the classroom.

Clear the display and memory before starting a calculation

It is good practice to remove any displayed numbers and to clear the calculator's memory if this is to be used to store new values. The clear key may be a combined clear and clear entry key with [C/CE] or [CE/C] on it. Clearing the memory can involve a [CM] (clear memory) or [MC] (memory clear) key. Children are less likely to make errors if they get into the habit of clearing the display, and where appropriate the memory, before starting a new calculation. Always ask children to check if there are any 'left-over' numbers on their calculator before they start using it.

Correct a wrong entry by using the clear entry key

Most children will clear the display and repeat the calculation if they think that they have made an error. This is fine, but in a more complex calculation it is quicker to clear the most recent entry. Children should be taught how and when to use the clear entry or [CE] key and when it is more appropriate to clear everything and just start again. Get children into to the habit of using the [CE] key correctly rather than starting again every time they make an incorrect entry.

Store a value in the calculator's memory and retrieve it during a multi-step calculation

As children become more confident users of the calculator they can be taught how to use the calculator's memory. The objective that relates to the use of the memory is in Year 6 progression to Year 7, so many Year 6 children could acquire these skills. Storing and retrieving numbers can assist them with multi-step calculations. The calculator may have four keys associated with the memory: [CM] or [MC] to clear the memory, [RM] or [MR] to retrieve the number stored in the memory, [M+] to add the number entered to the number in the memory and [M–] to subtract the entered number from the number in the memory. Make sure children understand the function of each key and ensure that they do not inadvertently change the number in the memory by using the wrong key. Remind children that if they are in doubt they should clear the memory before they start a calculation.

Keep track of a calculation and record the method used

When using a calculator, children should be taught to record their calculations, together with the answers they obtain, at each stage in a multi-step calculation. They should be encouraged to check whether each answer makes sense as they work through a problem. Children who are confident in using the memory should still be asked to record the calculations involved. Children need to understand the difference between recording their method and recording the steps they go through on the calculator. Emphasise that recording the method is about recording the number sentences or the calculations involved.

Use of other function keys

Children may have access to more sophisticated calculators that have additional function keys, such as square root, square or power keys, and fraction notation. Knowledge of how to use these keys can provide children with the opportunity to apply and extend their mathematics. The Year 6 progression to Year 7 includes the use of the square root key. For example, multiplying a number by itself or using the square or power key to generate the square numbers can involve identifying square numbers well beyond 100. Finding squares and then square roots demonstrates the inverse operation to children. Finding the positive square root of a non-square number such as 7, then re-entering the number displayed and squaring it, shows children the way calculators round their displays.

B. The mathematical understanding and interpretation skills needed to use a calculator to support learning

Recognise the likely size of the answer and check answers

Children recognise that calculation is a precise skill – there is only one correct answer. To be good at it requires a good understanding and knowledge of number. While children may believe the calculator is a precise tool, remind them that the calculator only responds to numbers entered and the keys pressed. Accuracy is required when it comes to entering values into the calculator. Errors occur when the wrong digit is entered or a key is not used correctly. When using a calculator, children must be reminded that they should always cast a questioning eye over their results. Children should be encouraged to use the checking strategies identified in the Framework's objectives, such as approximating, looking at the most and least significant digits, checking the number of digits in the answer, monitoring the position of the decimal point or carrying out the inverse operation.

Recognise negative numbers in the display

Children are introduced to negative numbers in Year 4 and use them in context. Errors in a subtraction calculation may lead to a negative number being displayed on the calculator. Children need to recognise when a number displayed is negative and not to simply ignore the sign. They should go back and check if a negative value was expected and makes sense. The calculator provides a useful tool for work around negative numbers. Year 6 children learn to find the difference between a positive and a negative integer and between two negative integers, again in a context that gives meaning to the numbers involved. Using a calculator for these calculations should be treated with caution as the manipulation of positive and negative integers can easily be misinterpreted. Using an image, such as a number line, is more reliable. For example, finding the difference between –3 °C and +4 °C could be misrepresented as 1 degree rather than 7 degrees if the – and + signs are used as operations. The context, too, can change the sign in the answer. The answer to the question: 'What is the change in temperature from –3 °C to 4 °C?' is: 'An increase of 7 degrees (or +7 degrees).' The answer to: 'What is the change in temperature from 4 °C to –3 °C?' is: 'A fall of 7 degrees or (–7 degrees).'

Enter and interpret money and measurement calculations

Children need to understand when and why the decimal point can disappear and can move about in the display. When £0.50 is entered, the number displayed is likely to be 0.5 as trailing zeros are not shown in decimal numbers. Multiplying 0.5 by 2 results in the number 1 being displayed and the decimal point has gone. Interpreting the results of a calculation often causes difficulties, for example 5.6 could mean £5.60, or 5 metres and 60 centimetres, or 5 kilograms and 600 grams, and so on. Children should be taught why and when it is important to convert all measurements into the same units before they carry out a calculation.

Calculations that involve time

Calculations that involve time are best not carried out with a calculator. Children will find it easier to use a clock face or a time line to do such work. This is less prone to error than using a calculator. Finding the interval of time on a journey that starts at 08:38 and ends at 14:19 does not involve the subtraction of decimal numbers and children too often fall into this trap. Using a calculator for time calculations should be avoided.

Carry out calculations with more than one step

Over time, children will use the calculator to carry out increasingly complex calculations, such as finding three eighths of a quantity or sharing equally the sum of four quantities between three. Using a calculator effectively, children need to know which operations to carry out first. Some calculators have brackets that children can use to help sequence the order of the operations. Entering a calculation for which the order of calculation is not left to right, such as £138.45 – (£8.24 × 6), can easily lead to errors. Most children should be encouraged to carry out the calculation in stages. Children should be taught how to select the correct sequence of operations in calculations that involve more than one step and record these for reference and checking.

Recognise and interpret rounding errors

Calculators generally work with more digits than those displayed. The numbers are often rounded before being displayed, rather than simply truncated at the point of display. This can sometimes lead to a build-up and magnification of errors, though this is unlikely to affect most of the calculations the children undertake. For example, if you divide 1 by 11, the answer displayed is 0.090909. Now multiply by 2, then by 4 and finally by 11. The answer should be 8 as we multiplied one eleventh by 2, 4 and 11. If there has been rounding at stages the number displayed will not be 8. There are situations in which the children need to interpret the number displayed. For example, sharing a sum of money may result in a number with three or more decimal places being displayed. In some situations the context will determine if the number is to be rounded up or down. Children should always ask if the number displayed makes sense in the context of the calculation or the problem.

Use the division operation to enter a fraction

Children should be taught how to enter fractions using the division operation and to recognise the decimal equivalent displayed. For example, when $^{3}/_{4}$ is entered as 3 [÷] 4, the number displayed is 0.75, its decimal equivalent. The calculator is a useful tool for children to establish that all tenths have equivalent decimal representations as numbers with one decimal place, while all hundredths have equivalent decimal numbers with one or two decimal places. Children should be taught to recognise decimal representations of familiar fractions and be able to convert one representation into the other.

Recognise recurring decimals

Using the calculator children will discover that some fractions entered on it will fill the display and often exhibit repeating patterns in the decimal digits displayed. They should recognise the decimal representation of some of the fractions they are familiar with. Many of these fractions, for example one third, have recurring decimal representations. The calculator offers children the opportunity to explore the decimal number patterns displayed.

Decide when a calculator is an appropriate tool to use

Children are usually given access to the calculators for a particular task. They recognise that on that occasion they are allowed to use them. Children need to recognise that any tool is designed for a purpose and the calculator is no exception. They should be given the opportunity to decide for themselves when a calculator might be helpful and to discuss when mental or written methods are more effective and efficient. Showing children that they have the knowledge and skills that enable them to calculate in less time than it takes them to enter the calculation into a calculator is a useful ploy. Spend time with the children discussing how the calculator supported the activity and what the children learned as a result of the activity.

Overview of calculator skills in Years 4, 5 and 6

What children should know how to do by the end of Year 4

Clear the display before starting a calculation

Children are less likely to make errors if they clear the display as a matter of habit before starting a new calculation.

Correct mistaken entries by using the clear entry key

Children tend to trust the calculator, which 'never goes wrong', without thinking about the possibility that they have made the mistake when entering numbers. Most children will clear the display and repeat the calculation if they think that they have made an error. They need to learn how and when to use the [CE] key.

Carry out one- and two-step calculations that involve all four operations

Most children have little difficulty with entering a one-step calculation to work out, for example, £4.55 [×] 17. However, when they are solving word problems, children are not always clear which values and what operations to use. They may misinterpret the question and enter the wrong calculation using the wrong values. It is good practice to get them to write down the calculations involved so they can check they have used the right operations and numbers.

Interpret the display correctly, particularly in the context of money

Children need to be taught how to interpret the displayed numbers, particularly large numbers as there no gaps to help them read it correctly. Decimal numbers can cause confusion when there is only one decimal place and the value has to be put into a context such as money.

Recognise negative numbers and use the sign-change key

Children may miss the minus sign that indicates a negative number. This usually appears on the extreme left of the display. It will appear if a subtraction calculation has been entered in the wrong order, for example.

What children should know how to do by the end of Year 5

Estimate the likely size of the answer and check answers appropriately

This is an important skill – errors in making entries often lead to answers that are nonsense, particularly when decimals or fractions are involved. Using some checking strategies, such as rounding and making an estimate or carrying out the inverse operation, will help children to avoid such errors.

Carry out measurement calculations and interpret the answer

Entering decimal numbers to carry out calculations that involve measurements can cause difficulty. It is not always clear if the decimal point has been entered until other entries are made and the numbers start moving along the display. For example, 5.6 could mean 5 metres and 60 centimetres, or 5 kilograms and 600 grams, and so on. In addition, children need to be taught to change all measurements to the same units before they do a calculation. This is best done manually before the values are entered.

Solve problems involving fractions

To find $^{3}/_{4}$ of 260 g children need to be taught that this calculation is represented by the calculation 260 [×] 3 [÷] 4, or recognise that the decimal equivalent of $^{3}/_{4}$ is 0.75 and use the calculation 0.75 [×] 260. These approaches support children's understanding of how a fraction is used as an operator and the equivalent representations of number as a fractions or a decimal. The use of fraction keys that are available in some calculators is likely to cause confusion and should be avoided.

What children should know how to do by the end of Year 6

Solve problems involving multi-step calculations

Children need to be familiar with the order of operations so that they choose the correct sequence in calculations that involve more than one step. They also need to practise jotting down parts of a calculation as they go along. Calculations such as: 8 × (37 + 58), 43 per cent of £285, or $^{3}/_{8}$ of 980 km are all multi-step and need to be taught and practised. At Key Stage 2 there should be no particular need to use a calculator to work out percentages, but if necessary the percentage can be represented as a fraction or decimal. So 43 per cent of £285 can be calculated as 285 [×] 0.43, or as 285 [×] 43 [÷] 100. Basic calculators usually have a memory. While there is no requirement for children to use the memory at Key Stage 2, children in Year 6 will probably enjoy learning to use it.

Recognise rounding errors

Although rounding errors are rare on modern calculators, they can still occur. Children need to know when answers are likely to have been rounded. For example, using the calculator to explore decimal representations of fractions will show that $^{1}/_{9}$ is represented by 0.1111111, but when this number is multiplied by 3 and 3 again the answer displayed may be 0.9999999, not 1.

Recognise recurring decimals

Children should be familiar with decimals representations such as $\frac{1}{3}$ and 0.3333333. They also need to recognise that not all the digits may recur in a decimal representation of a fraction, as in $\frac{1}{6}$, with the decimal equivalent 0.1666666, or $\frac{1}{7}$, which is 0.1428571 with the six digits 142857 recurring.

Use brackets, the memory and square root key

Children who have a good understanding of number and who are confident with the calculation aspects identified above may explore a range of extra facilities offered by the calculator. The memory is useful when undertaking multi-step calculations or generating more complex sequences, brackets help demonstrate and cope with the order of operations and the square root key opens the door to exploring a new set of non-recurring numbers. These skills might be developed after the end-of-year tests to support extended work and investigative activity or might be introduced earlier to children who are on course to attaining level 5.

Appendix: Objectives

Using and applying mathematics	Calculating
Year 4 Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate	**Year 4** Use a calculator to carry out one-step and two-step calculations involving all four operations; recognise negative numbers in the display, correct mistaken entries and interpret the display correctly in the context of money
Year 5 Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use	**Year 5** Use a calculator to solve problems, including those involving decimals or fractions (e.g. find $^{3}/_{4}$ of 150 g); interpret the display correctly in the context of measurement
Year 6 Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use	**Year 6** Use a calculator to solve problems involving multi-step calculations
Year 6 progression to Year 7 Solve problems by breaking down complex calculations into simpler steps; choose and use operations and calculation strategies appropriate to the numbers and context; try alternative approaches to overcome difficulties; present, interpret and compare solutions	**Year 6 progression to Year 7** Use bracket keys and the memory of a calculator to carry out calculations with more than one step; use the square root key