Guidance paper - Using and applying mathematics

Background and context

The renewal of the Primary Framework for mathematics has provided an opportunity to take a fresh look at the overall structure of the primary curriculum for mathematics. This has resulted in a reorganisation of the objectives, now structured into seven core areas of learning in mathematics or broad strands. These strands help to identify key aspects of mathematics that inform teaching and learning. They relate directly to the 1999 *Framework for teaching mathematics from Reception to Year 6* and the 2000 National Curriculum programmes of study.

The seven strands are:

- *Using and applying mathematics*
- *Counting and understanding number*
- *Knowing and using number facts*
- *Calculating*
- *Understanding shape*
- *Measuring*
- *Handling data*

This paper offers additional detail and guidance on how the *Using and applying mathematics* strand of the electronic Framework is structured and how it might be embedded in the whole of the primary mathematics curriculum.

Links to the National Curriculum programmes of study for mathematics and the 1999 Framework

The table below shows how these seven strands in mathematics link to the National Curriculum programmes of study and the strands in the 1999 *Framework for teaching mathematics from Reception to Year 6*.

Core areas of learning in mathematics	Key Stage 1 and 2 programmes of study	Strands in the 1999 Framework
• Using and applying mathematics	• Using and applying number • Using and applying shape, space and measures • Using and applying handling data	• Making decisions • Reasoning and generalising about numbers and shapes • Problems involving 'real life', money or measures
• Counting and understanding number	• Numbers and the number system	• Counting, properties of number and number sequences • Place value and ordering, including reading and writing numerals and estimating and rounding • Fractions, decimals, percentages, ratio and proportion
• Knowing and using number facts	• Calculations • Solving numerical problems	• Rapid recall of addition and subtraction facts • Rapid recall of multiplication and division facts • Checking the results of a calculation
• Calculating	• Calculations	• Understanding addition and subtraction • Understanding multiplication and division • Mental calculation strategies • Pencil and paper procedures • Using a calculator
• Understanding shape	• Understanding patterns and properties of shape Key Stage 1 • Understanding properties of shape Key Stage 2 • Understanding properties of position and movement	• Shape and space
• Measuring	• Understanding measures	• Measures
• Handling data	• Processing, representing and interpreting data	• Organising, using, interpreting and handling data

When the *Framework for teaching mathematics from Reception to Year 6* was published in 1999, as the above table shows, its focus was very much on number and calculations. The 1999 Framework has objectives under the broad heading of *Solving problems*, subdivided into three sections:

- Making decisions
- Reasoning and generalising about numbers and shapes
- Problems involving 'real life', money or measures.

However, the National Curriculum gave more attention to *Using and applying mathematics*. This was built into each of:

- Ma2 Number (Key Stages 1 and 2)
- Ma3 Shape, space and measures (Key Stages 1 and 2)
- Ma4 Handling data (Key Stage 2)

In an appendix at the end of the paper, the 'using and applying' sections from Ma2, Ma3 and Ma4 are included for reference.

Using and applying mathematics in the electronic Framework

One of the aims of the Primary Framework is to give greater attention to using and applying mathematics. The *Using and applying mathematics* strand has five themes and there is progression built into each theme.

The five themes are:

- Solving problems
- Representing – analyse, record, do, check, confirm
- Enquiring – plan, decide, organise, interpret, reason, justify
- Reasoning – create, deduce, apply, explore, predict, hypothesise, test
- Communicating – explain methods and solutions, choices, decisions, reasoning

These themes relate directly to the three subdivisions of 'using and applying' in the National Curriculum programmes of study. The two themes, representing and enquiring, have been separated to emphasise their importance in mathematics at every stage. For example, when children move from combining groups of objects in a practical context to manipulating the number symbols, they are effectively representing a range of possible situations. This underlying idea of modelling situations with mathematics is an important theme but receives relatively little attention in the 1999 Framework.

In mathematics, there are processes that are particularly important and one of these is pursuing a line of enquiry. This involves children in making decisions and organising and interpreting information and results. This relates to the 1999 Framework's solving problems subheading *Making decisions* and picks out the investigative element in the reasoning subdivision of using and applying in National Curriculum programmes of study.

While identified separately, the five themes are very much linked. When children solve problems or follow a line of enquiry, they will be representing their ideas, using numbers, symbols or diagrams; they will be involved in reasoning and predicting and communicating their results, orally or in writing. The themes are intended to provide a tool to outline and track progression. Below is a table showing the progression within each of the five themes in the Primary Framework's *Using and applying mathematics* strand.

Using and applying mathematics	**Solving problems**	**Representing**	**Enquiring**	**Reasoning**	**Communicating**
Foundation Stage	Use developing mathematical, ideas and methods to solve practical problems	Match sets of objects to numerals that represent the number of objects	Sort objects, making choices and justifying decisions	Talk about, recognise and recreate simple patterns	Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices

Using and applying mathematics	Solving problems	Representing	Enquiring	Reasoning	Communicating
Year 1	Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change'	Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context	Answer a question by selecting and using suitable equipment, and sorting information, shapes or objects; display results using tables and pictures	Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions	Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures
Year 2	Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence	Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem	Follow a line of enquiry; answer questions by choosing and using suitable equipment and selecting, organising and presenting information in lists, tables and simple diagrams	Describe patterns and relationships involving numbers or shapes; make predictions and test these with examples	Present solutions to puzzles and problems in an organised way; explain decisions, methods and results in pictorial, spoken or written form, using mathematical language and number sentences
Year 3	Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations	Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure	Follow a line of enquiry by deciding what information is important; make and use lists, tables and graphs to organise and interpret the information	Use patterns and relationships involving numbers or shapes, and use these to solve problems	Describe and explain methods, choices and solutions to puzzles and problems, orally and in writing, using pictures and diagrams

Using and applying mathematics	Solving problems	Representing	Enquiring	Reasoning	Communicating
Year 4	Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate	Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem	Suggest a line of enquiry and the strategy needed to follow it; collect, organise and interpret selected information to find answers	Identify and use patterns, relationships and properties of numbers or shapes; investigate a statement involving numbers and test it with examples	Report solutions to puzzles and problems, giving explanations and reasoning orally and in writing, using diagrams and symbols
Year 5	Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use	Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem	Plan and pursue an enquiry; present evidence by collecting, organising and interpreting information; suggest extensions to the enquiry	Explore patterns, properties and relationships, and propose a general statement involving numbers or shapes; identify examples for which the statement is true or false	Explain reasoning using diagrams, graphs and text; refine ways of recording using images and symbols
Year 6	Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use	Tabulate systematically the information in a puzzle or problem; identify and record the steps or calculations needed to solve it, using symbols where appropriate; interpret solutions in the original context and check their accuracy	Suggest, plan and develop lines of enquiry; collect, organise and represent information, interpret results and review methods; identify and answer related questions	Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of *c* pens at 15 pence each is 15*c* test pence)	Explain reasoning and conclusions, using words, symbols or diagrams as appropriate

Using and applying mathematics	Solving problems	Representing	Enquiring	Reasoning	Communicating
Year 6 progression to Year 7	Solve problems by breaking down complex calculations into simpler steps, choose and use operations and calculation strategies appropriate to the numbers and context; try alternative approaches to overcome difficulties; present, interpret and compare solutions	Represent information or unknown numbers in a problem, for example in a table, formula or equation; explain solutions in the context of the problem	Develop and evaluate lines of enquiry; identify, collect, organise and analyse relevant information; decide how best to represent conclusions and what further questions to ask	Generate sequences and describe the general term; use letters and symbols to represent unknown numbers or variables; represent simple relationships as graphs	Explain and justify reasoning and conclusions, using notation, symbols and diagrams; find a counter-example to disprove a conjecture; use step-by-step deductions to solve problems involving shapes

Embedding Using and applying mathematics in all strands

To be successful, *Using and applying mathematics* has to be embedded in the teaching and learning of mathematics. Over time, the approach taken must help children to see how the routines and individual steps they have learned in the context of different stands can be combined into 'linked chains' of calculations, decisions, reasoning and communication. This requires practice and takes time to learn. For some children, working through sets of similar questions, puzzles and problems may be a struggle, but it is an essential step in being able to recognise how to solve problems of a particular type. When learning is successful children begin to use the 'chains' of reasoning they recognise and are able to create their own, teasing out solutions to non-routine questions and problems.

Of course, teaching children how to use and apply their mathematics is not routine. There are a number of structured approaches to problem solving that have been devised to help children to identify the steps and stages they might work through. However, many of these prompt children to read the question carefully, identify the information presented, and so on. While these are a helpful checklist for children, they do not always apply and they do not help children to select the strategy needed to solve the problem. Teaching children to use and apply their mathematics involves careful selection and dissection of questions, problems and tasks; these should be of a particular type to provide children with practice so they can develop the confidence they need to sustain what can often be a struggle to find a solution. The skills of using and applying mathematics involve the organisation of thinking, the selection of ideas and strategies to try and the confidence to determine if these ideas and strategies will actually work.

The five themes in the Using and applying mathematics strand

1. Solving problems

Formatted: Bullets and Numbering

Solving problems lies at the heart of mathematics. The definition of numeracy set out in the 1999 Framework recognised this and defined numeracy as a proficiency that requires 'an inclination and ability to solve number problems in a variety of contexts', resulting in children 'who are confident enough to tackle mathematical problems without going immediately to teachers or friends for help'. Acquiring the confidence and skills involved takes time and requires practice and success.

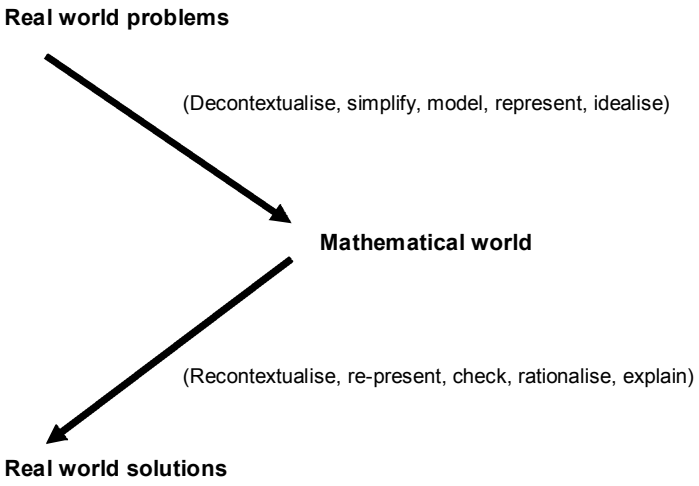
Children need to solve problems to become problem solvers. This may be stating the obvious, but it implies that children have to be given the time and space to tackle problems in mathematics lessons if they are to be confident and competent problem solvers. Problem solving should be integrated into mathematics teaching and learning, and become a regular part of the children's work. Devoting specific lessons to problem solving will help but embedding them into everyday lessons will provide the frequent and regular practice and consolidation children need. Problem solving should not be seen as a 'Friday-only' activity.

In the renewed Primary Framework the problem-solving theme focuses on problems that involve calculations set in wider-ranging contexts as the children become more skilled and confident. The progression highlights the increasing complexity of the problems the children tackle as they move from one-step to multi-step problems and begin to meet those problems that are more complex and where less routine approaches are needed to solve them.

2. Representing

Formatted: Bullets and Numbering

The nature of mathematics is that it provides an uncluttered and generally consistent 'world' within which to work and think. Solving problems involves moving between the 'real world' and the 'mathematical world'. When children recognise that a problem such as: 'How much will seven oranges cost if four oranges cost £1?' can be represented by the calculations $100 \div 4 = 25$ and $25 \times 7 = 175$ they are able to move from the 'real world' and work in the 'mathematical world'. The calculations involve just numbers. There is no need to refer to money or oranges. It is only when we use and interpret the number 175 to decide that the cost of seven oranges is 175p or £1.75 that we move back into the 'real world'.



Moving from the 'real world' to the 'mathematical world' requires an initial analysis of available information. This is to determine what might be discarded or put on hold and what is immediate and essential. It involves having a go to see if the chosen information and pathways are sensible and if they indicate next steps and directions to follow. This process may take time and, in most cases, involves some trial and improvement. However, it is through this process that children evaluate potential strategies and chains of reasoning are formed. It is where the breakthrough is made or blind alleys emerge. It is where the National Strategy's reference to inclination to solve problems is evidenced and developed.

Once a decision has been made and an appropriate strategy identified, children can begin to record their ideas, moving into the 'mathematical world' to do so. Their pictures, diagrams, lists, tables or calculations become their working tools. Children use these tools, drawing on their mathematical knowledge, skills and understanding, to do the necessary mathematics and arrive at a solution. They then return to the 'real world' to check if their results make sense within the context of the problem.

This process of selecting the key bits of information that are needed and representing the problem, using mathematical calculations, tables or diagrams, lays the foundation for the process referred to as 'mathematical modelling'. This is essentially re-presenting a problem, for example, as a series of mathematical statements or pictures that can be manipulated, refined and solved away from the context of the problem. So, when young children correctly record tally marks to count, draw block graphs to find most frequent outcomes or use number sentences to represent some practical problem, they are beginning their journey down the road as mathematical modellers.

The processes described above apply to the solution of mathematical or logical puzzles too. The analysis of written information, pictures or diagrams, leads to some alternative representation or structure. A Carroll or Venn diagram can help to sort the information; a table or list may be a way of presenting all possibilities from which the impossible or anomalous cases can be identified and removed. The representation is a tool to aid thinking and the choice of representation is important in highlighting what does or does not meet the conditions or criteria that define the puzzle.

3. Enquiring

Learners are engaged by successful and exciting learning. They become involved in finding out for themselves, asking and answering questions of their own and sharing what they have discovered with others. The posing of questions is an element of creativity. It starts the process needed for the generation of ideas that are new to the learner, which takes them on a route of discovery.

This theme in the *Using and applying mathematics* strand involves children in following a line of enquiry. Initially the questions the children pursue may be given to them. As the children become more skilled at planning and organising their strategies and thoughts, and more confident at sustaining such activity, new questions will arise. These can be gathered, discussed and refined so that the children can pursue those that appeal to them.

The lines of enquiry may be drawn from any of the other six strands and, depending on the questions posed, involve varying degrees of decision making and reasoning. What is important is that children make informed decisions that they can justify, and begin to sustain a line of reasoning in support of their steps in the enquiry. The children need to be taught how to use pictures, lists, tables, graphs and diagrams to help them to organise and interpret any information they collect.

4. Reasoning

Formatted: Bullets and Numbering

Children need to be taught how to describe, interpret and explain what they see and how to use this as a basis to inform their thinking and reasoning. Making deductions on the basis of given information, or from particular situations, is an extension of that information or situation. Consequently, the first step is to be able to establish the starting point, being clear about the

properties, relationships and structures that are present. Children are often too quick to make assumptions about what they *think* they see rather than critically appraise the information, context and situations they are *given*.

Children need to be taught how to record their thinking and reasoning in mathematics as they describe, replicate and create patterns and explore properties and relationships. These skills of recording, using objects, pictures, numbers or shapes, help children to see what *is* and begin to consider what *might be*. They help children to clarify what is the same and what is different. They help children to collect evidence when testing general statements or to predict and propose new hypotheses.

While making and using pictures, diagrams and notes to aid thinking are important skills for children, making decisions and deductions form another set of skills that children need to learn. This theme in the *Using and applying mathematics* strand involves children in learning to acquire these skills. Identifying patterns, properties and relationships in number, measures or shape and using them to make sensible deductions based on clear reasoning is a valued skill.

Reasoning involves some understanding of 'logical rules': knowing what and when things are the same or different; recognising what equivalence means and how to express, for example, equivalent number sentences or calculations; understanding implication so that certain properties imply others hold too, such as multiples of 6 are also multiples of 2 and 3 or a quadrilateral with four right angles means its opposite sides are equal. Part of this is having a good grasp of the mathematical vocabulary and language of explanation and reasoning. Using this language, children can begin to record their explanations and reasons and refine these, using diagrams, graphs and mathematical notation and symbols.

Keeping track of thinking or the steps involved in solving problems or puzzles, for example, when carrying out calculations with a calculator or examples that have been tested when investigating a statement involving numbers or shapes, requires some systematic recording. This is often helped by organising notes into a table, diagram or structured list. Children need to see working examples and models that they can use, supported with opportunities to share and evaluate one another's approaches. Together, these provide children with the tools from which to make informed choices about how best to organise their work.

There is also the language children use to explain and reason. They need to hear the language and vocabulary of reasoning so they can use it themselves. Learning how to use this is less about acquiring a new vocabulary, more about practising how to express thoughts in a coherent and logical way. This process requires an audience. Discussion, in pairs, small groups or the whole class, provides children with different forums for practising and refining the language used and for getting feedback from peers or adults.

As children's oral skills develop they begin to draw on pictures, diagrams or statements to help to record their explanations and reasons, and employ mathematical notation and symbols to help them. To explain how to complete a logic puzzle involving statements about five people seated in a row may involve written instructions; to explain that the 51st number in the sequence 2, 7, 12, … is 252 as it is $2 + 50 \times 5$ draws on mathematical notation; to explain why a particular diagram cannot be the net of a given solid may involve making new or annotated diagrams. To reason that there are only three ways to use 1p and 2p coins to give the exact money for a sweet costing 5p may involve listing all possibilities as calculations; to reason that 203 is not a multiple of 4, as 20 is a multiple of 4 and so 200 is a multiple of 4 uses statements and symbols; to reason that you cannot draw an equilateral triangle joining points on a 3 by 3 square grid may involve a series of pictures or an explanation that draws on lengths of lines.

5. Communicating

Communication can be personal or to a small and particular audience. In the classroom some recorded work may only ever be seen by the child or the teacher. This communication is, however, an important part of learning in mathematics and the associated skills need to be developed and applied.

As we use and apply our mathematics or explore in the 'mathematical world', we record our thoughts to help us with the process and to keep track of the direction we are taking. The records we keep may be ad hoc or just annotations on a diagram or graph. They may be quite personal or idiosyncratic. What we do record should support our thinking; it needs to be clear communication to ourselves so when we look back it makes sense. In essence the communication is personal and relates to the doing of the mathematics; it is part of the process and may not be intended for a wider audience.

In the classroom, everyday language is used to teach and learn mathematics. This is usually adequate for talking about the methods and processes being learned. Over time, some of this language takes on a special meaning and additional vocabulary is added to give names to shapes or the properties of numbers. The words *face*, *difference* or *regular* can signal special meanings in the mathematics classroom and children learn to talk about *reflections*, *products*, *cuboids* and *perpendicular* and *parallel* lines. Alongside this extra vocabulary children learn how mathematics employs language, for example, to describe quantities and relationships – *approximately 50*, *rounded to the nearest 10*, *greater than*, *a quarter of*, *is a multiple of* and *a reflection of*.

Returning to earlier work from time to time encourages children to interpret their personal communication and to see how effective it was in relaying their thoughts and ideas. This is part of the wider note-taking skills children need to develop to support their learning across the curriculum. It begins to foster ideas that what is recorded, however personal, is not to be forgotten and lost, but provides information for later use, to recall and revise earlier knowledge and ideas.

In their everyday mathematics lessons, while children may recognise and use the pictures, diagrams and symbols involved in the above explanations and reasoning, they struggle to apply them when asked to frame an explanation or reasoning of their own. This process requires those additional skills and the understanding learned through discussion and negotiation with others. Building into lessons the opportunity to discuss, build and negotiate explanations and reasoning with other children is important. They may be quite brief activities, as part of the main teaching or a plenary, to recap on or consolidate learning: the ability to communicate mathematical thinking, explanations and reasoning to others is only learned through guided practice. Regular and frequent practice at using and applying mathematics needs careful planning to identify purposeful activity that can be monitored so that children's progress can be assessed to inform their future learning.

Appendix: *Using and applying mathematics* in the National Curriculum programmes of study

Key Stage 1	Problem solving	Reasoning	Communicating
Ma2 **Number**	• Approach problems involving number, and data presented in a variety of forms, in order to identify what they need to do • Develop flexible approaches to problem solving and look for ways to overcome difficulties • Make decisions about which operations and problem-solving strategies to use • Organise and check their work	• Present results in an organised way • Understand a general statement and investigate whether particular cases match it • Explain their methods and reasoning when solving problems involving number and data	• Use the correct language, symbols and vocabulary associated with number and data • Communicate in spoken, pictorial and written form, at first using informal language and recording, then mathematical language and symbols
Ma3 **Shape, space and measures**	• Try different approaches and find ways of overcoming difficulties when solving shape and space problems • Select and use appropriate mathematical equipment when solving problems involving measures or measurement • Select and use appropriate equipment and materials when solving shape and space problems	• Recognise simple spatial patterns and relationships and make predictions about them • Use mathematical communication and explanation skills	• Use correct language and vocabulary for shape, space and measures

Key Stage 2	Problem solving	Reasoning	Communicating
Ma2 **Number**	• Make connections in mathematics and appreciate the need to use numerical skills and knowledge when solving problems in other parts of the mathematics curriculum • Break down a more complex problem or calculation into simpler steps before attempting a solution; identify the information needed to carry out the tasks • Select and use appropriate mathematical equipment, including ICT • Find different ways of approaching a problem in order to overcome any difficulties • Make mental estimates of the answers to calculations; check results	• Understand and investigate general statements • Search for pattern in their results; develop logical thinking and explain their reasoning	• Organise work and refine ways of recording • Use notation, diagrams and symbols correctly within a given problem • Present and interpret solutions in the context of the problem • Communicate mathematically, including the use of precise mathematical language
Ma3 **Shape, space and measures**	• Recognise the need for standard units of measurement • Select and use appropriate calculation skills to solve geometrical problems • Approach spatial problems flexibly, including trying alternative approaches to overcome difficulties • Use checking procedures to confirm that their results of geometrical problems are reasonable	• Use mathematical reasoning to explain features of shape and space	• Organise work and record or represent it in a variety of ways when presenting solutions to geometrical problems • Use geometrical notation and symbols correctly • Present and interpret solutions to problems

Ma 4 **Handling data**	• Select and use handling data skills when solving problems in other areas of the curriculum, in particular science • Approach problems flexibly, including trying alternative approaches to overcome any difficulties • Identify the data necessary to solve a given problem • Select and use appropriate calculation skills to solve problems involving data • Check results and ensure that solutions are reasonable in the context of the problem	• Explain and justify their methods and reasoning	• Decide how best to organise and present findings • Use the precise mathematical language and vocabulary for handling data