Renewing the Primary Framework for mathematics

Discussion paper

Mathematics and the primary curriculum

Background and context

This short discussion paper raises some questions and key points about the teaching and learning of mathematics in our primary schools. It is not intended to give all the answers to the questions it raises. Rather, it is to stimulate and inform a professional debate on how we might use the opportunity provided by the renewal of the 1999 *Framework for the teaching of mathematics from Reception to Year 6*, to strengthen provision in primary schools.

The renewal of the 1999 Framework builds on the successes that have been made to date. It provides the opportunity to review the practices that have been established since the National Numeracy Strategy was first introduced into schools and to refocus the drive to raise standards for all children. It is time to review expectations, entitlement, provision and practices in mathematics teaching and learning in the light of the gains made.

A key aim of the review is to engage all professionals who work with children in the primary classroom in a dialogue about the teaching and learning of mathematics. We want all our children to succeed in mathematics and more children are doing so. We want them to be excited by mathematics and to continue to study the subject well beyond their primary education. We want children to see how mathematics can help them become better equipped for future life. High expectations – but, given the gains we have made to date, it's time to aim for the stars.

Why is mathematics important?

A report into mathematics education, *Making Mathematics Count*, 2004, by Professor Adrian Smith, offers a number of reasons why mathematics is important. In one of the paragraphs the report outlines the importance of mathematics to modern society:

> *'Mathematics is of central importance to modern society. It provides the vital underpinning of the knowledge economy. It is essential in the physical sciences, technology, business, financial services and many areas of ICT. It is also of growing importance in biology, medicine and many of the social sciences. Mathematics forms the basis of most scientific and industrial research and development. Increasingly, many complex systems and structures in the modern world can only be understood using mathematics and much of the design and control of high-technology systems depends on mathematical inputs and outputs.'*
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> *Smith, 2004 Making Mathematics Count*

For most of us, our awareness of the role of mathematics in modern society is as a user of the end products rather than from a direct involvement in the process. Children seek the latest electronic goods and adults benefit from the

medical advances that result from those processes listed above. But the report also offers a more fundamental set of reasons for children and adults to learn mathematics.

> *'The acquisition of at least basic mathematical skills – commonly referred to as "numeracy"– is vital to the life opportunities and achievements of individual citizens. Research shows that problems with basic skills have a continuing adverse effect on people's lives and that problems with numeracy lead to the greatest disadvantages for the individual in the labour market and in terms of general social exclusion. Individuals with limited basic mathematical skills are less likely to be employed and, if they are employed, are less likely to have been promoted or to have received further training.'*
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> *Smith, 2004 Making Mathematics Count*

This more pragmatic viewpoint is one to which all primary teachers can subscribe. The primary curriculum is the point at which children's acquisition of these basic mathematical skills starts and, for most children, it is a very successful experience. They leave primary schools well equipped to progress and succeed in the future labour market. But this pragmatic view, while important, does not say anything about how a study of the subject might improve individual children and help them to become more rounded persons. Here is a final quote from the report.

> *'Mathematics provides a powerful universal language and intellectual toolkit for abstraction, generalisation and synthesis. It is the language of science and technology. It enables us to probe the natural universe and to develop new technologies that have helped us control and master our environment, and change societal expectations and standards of living. Mathematical skills are highly valued and sought after. Mathematical training disciplines the mind, develops logical and critical reasoning and develops analytical and problem-solving skills to a high degree.'*
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> *Smith, 2004 Making Mathematics Count*

Here, the justification for mathematics being important is in its ability to develop and support children's thinking, reasoning and problem-solving skills. The skills embedded in mathematics and the discipline of learning and using mathematics provides children with other cognitive skills that they can use across and beyond the school curriculum. The training received through the study of mathematics provides children with skills that are in high demand. The ability to analyse information and to solve problems are key skills embedded in the primary curriculum, within which mathematics has a significant role to play.

In summary, mathematics makes a significant contribution to modern society; the basic skills of mathematics are vital for the life opportunities of our children; and mathematics develops the mind and those highly valued cognitive skills.

So what is mathematics?

Mathematics is an old and well-established subject. It has many rich, historic legacies. Mathematics has been studied by most ancient cultures and its importance in modern society continues. Over time the focus of study has changed in line with the changing needs of society. Who would have predicted that the early number theory of the Greeks would inform the code-breaking work undertaken during the Second World War, or become a tool used on credit-card transactions to strengthen security in the financial world?

- **Mathematics builds from simple definitions and propositions that are based on observation.**

For example, shapes that can be seen, touched and made in the real world are usually imprecise. Mathematics helps to define them and give them structure. The definition of parallel lines, equal angles and similar and congruent shapes has created a geometry in which to think and explore. Propositions about shapes have led to new observations and proven properties that designers and artists use in construction. The imagined world of precise shapes has influenced the world in which we live. Children enjoy living in imagined worlds and creating stories that take the real world to extremes, rather as mathematics has done down the ages.

- **Mathematics starts from a desire to explain the real world.**

For example, the number system used today started because of the need to record quantities of objects or events. Counting the passage of time in days or weeks led to the development of the calendar and the ability to predict the phases of the Moon and changes in tides. This informed our understanding of the movement of planets and, ultimately, led to the ability to put a man on the Moon. The natural inquisitiveness of children and our desire to explain the world around us is the start of this adventure.

- **Mathematics involves measuring, comparing and classifying objects.**

For example, local trading led to a need to standardise quantities to replace a system of bartering. The standardisation grew as access to new worlds extended trade. Mathematics provided a tool with which to quantify the standard measures and, from this, abstract the process of measuring that is now applied to quite disparate outcomes. Measures of chance and risk, measures of public opinion and measures of climate change are all developed from the application of mathematical ideas. Within mathematics there are measures of chaos, the infinite and discontinuities: all stimuli to a vivid imagination.

From an early age, children compare and classify items by smell, taste or colour. They begin to start to measure by comparison or by counting the numbers of objects; the playground is alive with the language of comparison, ‘more than’, and classification, ‘same as’, and counting games where the outcomes are measured and compared. This naïve use of mathematics reflects its development and historic legacy.

- **Mathematics describes patterns, properties and general concepts.**

For example, even the symbol for three (3) represents a general concept. It can be a label, peg number 3, a position, third in line, or a quantity, *Three Little Pigs*. The early identification of numbers that had particular properties led to a detailed study of number theory. Many mathematical hours have been spent trying to derive a formula to predict the prime numbers – but still without success. However, the search for such formulae has provided a language and symbolism in use today. There are odd, prime, square, triangular and perfect numbers. Mathematics, too, has built up a vocabulary to describe patterns and properties in shape, such as 'symmetric' or 'regular'. Children use the terms *reflection*, *rotation* and *translation* to describe pattern, movement and properties of shapes. They acquire general concepts such as *quadrilateral*, *equal to* or *perimeter* through experience, even though the general concept may not have been defined. Children's ability to extract the essential properties and generalise from particular cases is a key skill in mathematics.

- **Mathematics provides the tools to abstract and work in an imagined world.**

For example, counting objects, then recording the number and calculating 50% by dividing the recorded number by two moves from a real world activity into the mathematical world. The world that mathematics provides is a representation of the real world, where objects may be represented by symbols, diagrams or statements. To identify the whole numbers in a set as $19 < m < 81$ and $m \mid 2$, or the even numbers from 20 to 80 inclusive, is easier than listing the numbers. Having set out the numbers involved, the representative 'm' can be manipulated and controlled in the mathematical world. Telling children that this particular triangle is isosceles because it has two equal sides conveys the abstraction that all isosceles triangles have two equal sides. When children carry out mental calculations they are working in an abstract world of mathematics. Even using an empty number line still requires imagination. Children visualise a generalised solid shape from a 2-D representation and they can describe the properties of the 3-D shape from the image. Such visualisation and generalised thinking are essential skills in mathematics.

- **Mathematics is a creative subject in which ideas can be generated, tested and refined.**

For example, making observations about the properties shared by a group of objects, inducing general statements from particular cases and making deductions from evidence about the properties of shape are all creative activities. Euclid's geometry created a whole set of propositions and theorems and challenges involving the use of a pair of compasses and a ruler to make particular shapes such as 'squaring the circle'. Later, geometry took new directions, based on assumptions about parallel lines and, in the case of topology, abstracted the geometry in a world where measurement was involved. Like the underground map, the route is clear but no measurements can be taken. These developments resulted from someone asking the question: 'What if…?' Children like to ask 'How…?', 'Why…?' and 'What if…?' questions and to be directed to solutions or be given ideas they can use to find out the answers. Questioning assumptions and conclusions, and testing

to see when and why they are valid, is a cornerstone of mathematical thinking and something children enjoy.

Ten approaches to the teaching of mathematics

1. Plan and provide a balanced experience that incorporates the ***exploration, acquisition, consolidation and application*** of knowledge and skills, with opportunities to use, extend and ***test ideas, thinking and reasoning***.

2. Share the ***excitement of learning*** mathematics and capture children's imagination by showing them the unusual or unexpected; give children examples of numbers or shapes that have special or surprising properties; show children how mathematics can be used creatively to represent, measure, predict and extrapolate to other situations.

3. ***Model*** for children how to explore mathematics and look for patterns, rules and properties; ***direct and steer children's learning*** by providing examples that enable them to observe and identify the rules and laws and deduce for themselves when they apply; help children to describe, replicate and use patterns and properties; ensure that they meet both general applications of the rules and exceptions.

4. Give children opportunity to ***consolidate*** their learning; introduce frequent and regular periods of ***practice*** that are short, sharp and focused on children securing, with the necessary accuracy and precision, the mathematical knowledge, understanding and skills they have learned; ensure that they recognise how their learning builds on previous learning and help them to see connections; ensure that they feel appropriately supported and challenged by the work they are set.

5. ***Engage*** with children's thinking; give them sufficient ***time for dialogue and discussion and space to think*** about their ideas, methods and mathematical representations of the real world; focus on underlying concepts and processes with prompting and probing questions.

6. ***Demonstrate*** and promote the correct use of ***mathematical vocabulary*** and the interpretation and ***use of symbols, images, diagrams and models*** as tools to support thinking, problem solving, reasoning and communication.

7. Provide children with the ***well-directed opportunity to use and apply*** what they have learned to solve routine and non-routine problems; highlight any properties or patterns they identify or create and make connections to other work they have done; draw on their ideas and model approaches and strategies children can use to support a line of ***enquiry*** or to interpret or explain their results and methods, using their own approaches and strategies.

8. Teach children how to ***evaluate solutions and analyse methods***, deciding if they are appropriate and successful; help children to understand why some methods are more efficient than others; provide opportunities to compare and measure objects and identify the extent to which shapes and calculations are similar or different; develop

children's understanding and language of equivalence and deduction to support reasoning and explanation.

9. Periodically identify the knowledge, skills and understanding children acquire; pause and take stock to ***review children's learning*** with them; highlight the strategies and processes upon which they are able to draw; provide opportunities that allow children to make connections and show how ideas in mathematics relate, and how their learning can be applied to new aspects of mathematics.

10. ***Model*** with children how they identify, ***manage and review*** their own learning; highlight the ***learning skills*** they have acquired and used and draw out how these might be applied across the curriculum.

In summary, mathematics teaching should:

- provide children with a balance of exploration, acquisition, consolidation and application
- ensure that children experience the excitement of learning mathematics
- direct and steer children to explore, identify and use rules, patterns and properties and model this process
- build in frequent short and sharp periods of practice and consolidation
- engage with children's thinking, giving sufficient time for dialogue and discussion and space to think
- demonstrate the correct use of mathematical vocabulary, language and symbols, images, diagrams and models as tools to support and extend thinking
- give well-directed opportunities for children to use and apply their learning
- teach children how to evaluate solutions and analyse methods and understand why some methods are more efficient than others
- pause and take stock to review children's learning with them
- model with children how they identify their learning skills, and manage and review their own learning.

Questions

- How is mathematics perceived by staff and children in the school?
- Does the teaching of mathematics direct, steer and review children's learning and model for children how to learn and use mathematics?
- In mathematics lessons is there a good balance of exploration, acquisition, practice, consolidation and application and do children make connections?
- Is the teaching engaging and interactive and is there additional support for children who need it?
- Do children get the opportunity to experience the excitement of learning mathematics?
- Is there sufficient time for dialogue and is children's use of mathematical language well developed?
- Do children have access to images, models and symbols they can use to support their thinking and reasoning?
- Are children engaged in evaluation and self-review as part of their mathematics learning?

